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The Light Shift Effect in the Coherent Population Trapping Cesium Maser

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Abstract—The light shift, as observed in the coherent population trapping (CPT) maser, was investigated theoretically and experimentally. It was found that the light shift originates from the various sidebands that are present in the spectrum of the frequency-modulated laser used to observe the CPT phenomenon.

I. Introduction

In previous papers on the cesium maser [1]–[3] based on CPT [4], it was reported that the maser output frequency is a quadratic function of the laser frequency tuning relative to the frequency of the S to P transition. No explanations were given as to its origin.

The present paper addresses this question. It is shown that, although in principle the two radiation fields creating the CPT phenomenon do not cause a light shift, if they have the same intensity, their respective effect on the other transition in the Λ scheme must be considered. Furthermore, in the case that a modulated laser is used, all sidebands must be taken into account because of their detuning from the various optical transitions. It was found that the light shift contains two parts. One part is independent of the laser tuning; we call it the power light shift. The other part depends on the square of the laser frequency tuning relative to the maximum of the optical resonance line; we call it the quadratic light shift. It was found that the size of both contributions may be considerably reduced by adjusting the relative amplitude of the various sidebands through the laser modulation index.

II. Theory

In the following analysis, the approximate three-level scheme shown in Fig. 1 is used to represent the lower levels of the cesium atom. A buffer gas is used to reduce wall collisions as well as increase the transit time of the atoms across the laser beam. Optical resonance line widths as broadened homogeneously by the cesium buffer gas collisions are of the order of 700 MHz, and the $P_{3/2}$ hyperfine levels are not resolved.

The study is made by means of the experimental setup illustrated in Fig. 2. Coherent population trapping is realized by means of two coherent radiation fields created with a frequency-modulated laser diode. In all of the experiments reported, the cesium D_2 transition was used. The two first sidebands of the modulated laser resonate with transitions from the $6S_{1/2}$, $F = 3$ or 4 levels to the $6P_{3/2}$, $F = 3$ or 4 levels. These sidebands create a coherence oscillating at the difference frequency of the two first laser sidebands, and this coherence is accompanied by a magnetization oscillating at the same frequency. This magnetization creates a magnetic field in the cavity and excites one of its modes. The field created reacts back on the atoms causing stimulated emission, and the energy emitted is detected at the cavity-coupling loop. The output power is a function of the applied frequency difference between the two laser sidebands; a maximum of emission is observed for exact tuning of this frequency to the hyperfine frequency. Because the hyperfine frequency is affected by several frequency shifts, such as magnetic field shift, buffer gas collision shift, spin exchange frequency shift and light shift, the frequency of the observed maximum of emission is affected by these shifts. In practice, the frequency of the microwave source used to generate the sidebands is locked to this maximum. The present paper addresses the light shift affecting this maximum of emission.

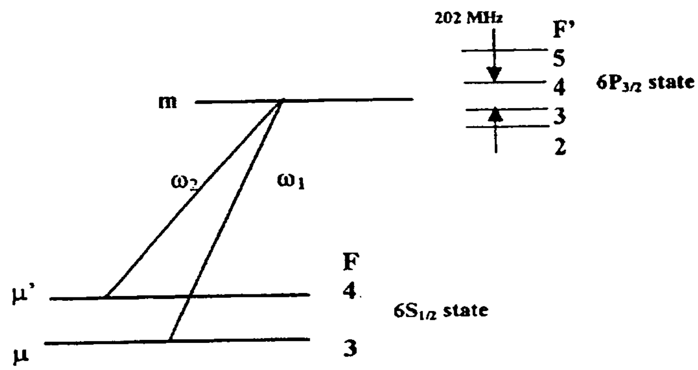

Fig. 1. Three-level system used to represent the ground state and the first excited state of the cesium atom used in the calculation.

The theory of the CPT maser developed in [1] shows that the residual light shift caused by the difference of intensity of the radiation field involved in the Λ scheme excitation is given by

$$\Delta\omega_{\rm LS} = -\frac{1}{4}\frac{\Delta_0}{(\Gamma^*/2)^2 + \Delta_0^2}\left(\omega_{1R}^2 - \omega_{2R}^2\right) \tag{1}$$

where ω_{1R} and ω_{2R} are the optical Rabi frequencies associated with the two laser radiation fields creating the CPT

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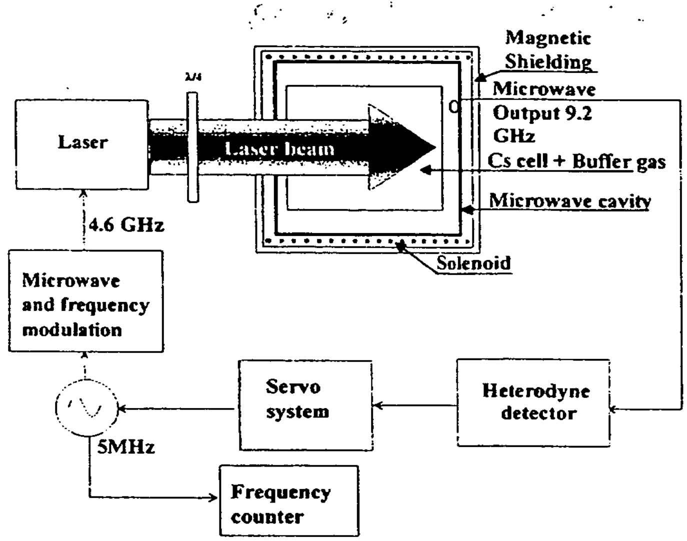

Fig. 2. Conceptual diagram of the CPT cesium maser.

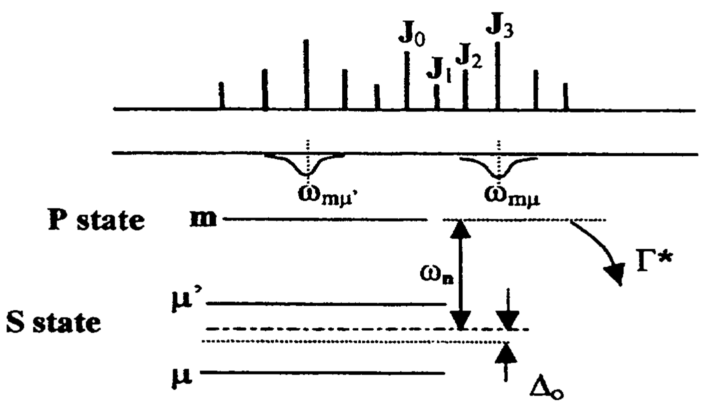

Fig. 3. Representation of the frequency modulated laser for $p = 6$.

phenomenon and Δ_0 is the detuning of the laser from the central frequency of the two transitions. In this model, the light shift is absent if the two Rabi frequencies are equal.

However, the laser radiation, at each of the transitions of the Λ scheme, interacts with the other transition and is out of resonance for that particular transition. Furthermore, in the present experiments, the laser is modulated at a frequency ν_{hf}/p, with $p = 2$, that is, 4.6 GHz. This technique produces sidebands at multiple frequencies of the modulation frequency and normally leaves a carrier. The amplitude of the carrier and of the sidebands depends on the modulation index m through Bessel functions of the first kind $J_n(m)$. All these components are non-resonant with the various transitions as illustrated in Fig. 3.

Although these sidebands do not cause real transitions they create a perturbation of the system and cause a light shift. This effect was not taken into account in the theoretical calculations reported in [1].

The angular frequency shift caused by a generic off-resonance laser radiation ω_L affecting each ground state hyperfine level is given in angular frequency units by [5], [6]:

$$\Delta\omega_\mu = \frac{1}{4}\,|\omega_{\mu R}|^2\,\frac{(\omega_L - \omega_{m\mu})}{(\omega_L - \omega_{m\mu})^2 + \Gamma^{*2}/4} \qquad (2)$$

$$\Delta\omega_{\mu'} = \frac{1}{4}\,|\omega_{\mu' R}|^2\,\frac{(\omega_L - \omega_{m\mu'})}{(\omega_L - \omega_{m\mu'})^2 + \Gamma^{*2}/4} \qquad (3)$$

and the resultant light shift of the hyperfine frequency is given by

$$\Delta\omega_{LS} = -(\Delta\omega_\mu - \Delta\omega_{\mu'}) \qquad (4)$$

where ω_L is the angular frequency of the considered sideband and $\omega_{\mu R}$ is its associated Rabi angular frequency.

The total light shift can be calculated through a summation over all sidebands of the modulated laser. In the calculation, we make the approximation that the separation between a given sideband and a transition is large compared with the width of the optical resonance line. In the case of cesium studied in [1], this width is of the order of 700 MHz because of buffer gas interactions. The approximation is thus valid for a maximum value $p = 6$. In that case, the term $(\Gamma^*/2)^2$ in the denominator of (2), and (3) can be neglected. Levels $F' = 3$ and 4 of the $P_{3/2}$ excited state are the two levels contributing to the CPT phenomenon, levels $F' = 2$ and 5 not providing the necessary link in the Λ scheme because of transition probability rules. These levels are not resolved because of the buffer gas broadening. In the calculation, we thus assume that the atom can be represented by the simple three-level system of Fig. 1. The detuning Δ_0 is thus defined as in Fig. 3 in relation to the midpoint of the two levels μ and μ' or equivalently to level m, which approximates state $P_{3/2}$ broadened by the buffer gas. Assuming $\Delta_0 \ll 1/2\omega_{\mu'\mu}$, and defining ω_{RL} as the Rabi frequency associated with the laser radiation without frequency modulation, the re-

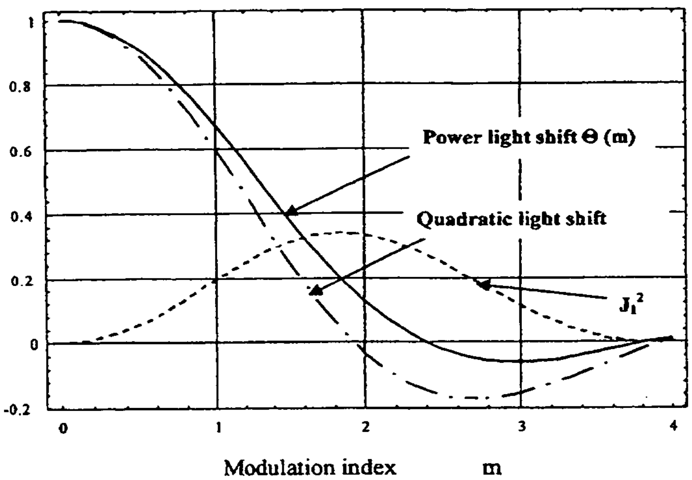

Fig. 4. Graphical representation of the power light shift and quadratic light shift coefficients in the CPT cesium maser as calculated on a three-level system for $p = 2$. The quadratic light shift coefficient, represented in the figure by the broken line, has been normalized to $\xi(m)/4$.

sult of the summation is

$$\frac{\Delta\omega_{LS}}{\omega_{\mu\mu'}} = \left(\frac{\omega_{RL}}{\omega_{\mu\mu'}}\right)^2 \left\{\Theta(m) + \xi(m)\left(\frac{\Delta_0}{\omega_{\mu\mu'}}\right)^2\right\} \quad (5)$$

where

$$\Theta(m) = J_0^2(m) + \frac{1}{2}J_{p/2}^2(m) - 2\sum_{\substack{n=1\\ n\neq p/2}}^{\infty} J_n^2(m)\frac{p^2}{(2n)^2 - p^2} \quad (6)$$

and

$$\xi(m) = 4J_0^2(m) + \frac{1}{2}J_{p/2}^2(m) - 8\sum_{\substack{n=1\\ n\neq p/2}}^{\infty} J_n^2(m)\frac{12n^2 + p^2}{((2n)^2 - p^2)^3}p^4. \quad (7)$$

The coefficients are plotted in Fig. 4 as a function of the index of modulation m for the case $p = 2$.

It is observed that these coefficients have zeroes as a function of the modulation index m. Thus, it is possible to adjust m to minimize the light shift. It is shown in Fig. 4 that for the zeroes of both functions Θ and ξ, the relative power of the first sideband J_1^2 is close to its maximum, a fact that is highly desirable for the excitation of the CPT phenomenon. Similar graphs can be obtained for $p = 4$ and $p = 6$. The values of the modulation index m for which the coefficients vanish are given in Table I.

III. Experimental Results

Measurements were done on the cesium CPT maser described in [1] and [2]. The cavity operated in the TE_{011} mode and had a loaded Q of the order of 3000. The buffer gas used in the cell was nitrogen at a pressure of 19 Torr. The cell length was 10 mm, and the laser beam diameter was of the order of 10 mm. The laser used had a nominal power output of 15 mW, which was adjusted to the desired levels by means of gray filters. The results are shown in Fig. 5 and 6. In these figures, the laser frequency detuning Δ_0 was measured from a laser frequency tuning that made the maser power output maximum. This frequency depended to some extent on the temperature of the maser Cs cell. This effect is believed to be due to the presence of the intense cycling transition $F = 4$ to $F' = 5$. This transition does not contribute to the CPT phenomenon but causes fluorescence that tends to reduce the hyperfine coherence. This effect is more important at higher cell temperatures because of the limiting quenching efficiency of the nitrogen buffer gas. Consequently, the measurements were made at a temperature of about 40°C, where we found the effect to be small.

TABLE I

Values of the Modulation Index for Which the Light Shift Coefficients Vanish and Value of Excitation Sideband $J_{p/2}^2$ for Which the Coefficient Θ Vanishes.

p	m for $\Theta = 0$	m for $\xi = 0$	$J_{p/2}^2 \Rightarrow \Theta = 0$
2	2.405	1.932	0.269
4	3.568	3.099	0.202
6	4.698	4.227	0.166

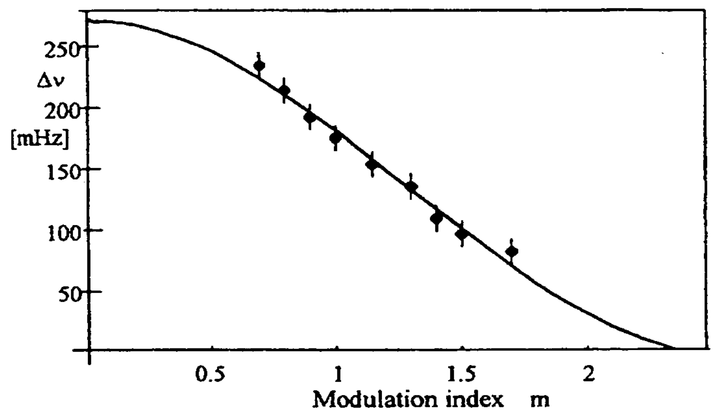

Fig. 5. Power light shift as observed experimentally. The solid curve is a best fit of (5) to the experimental data with $\Delta_0 = 0$.

A. Power Light Shift

The results of Fig. 5 were obtained for the following experimental conditions: laser modulation frequency, 4.6 GHz; modulation index, variable; and relaxation rate as measured by means of the free induction decay, $\gamma_2 = 330s^{-1}$ ($T_{cell} = 40$°C).

The laser power was set to obtain a maser line width equal to 2 kHz for a modulation index $m = 1.6$. In Fig. 5,

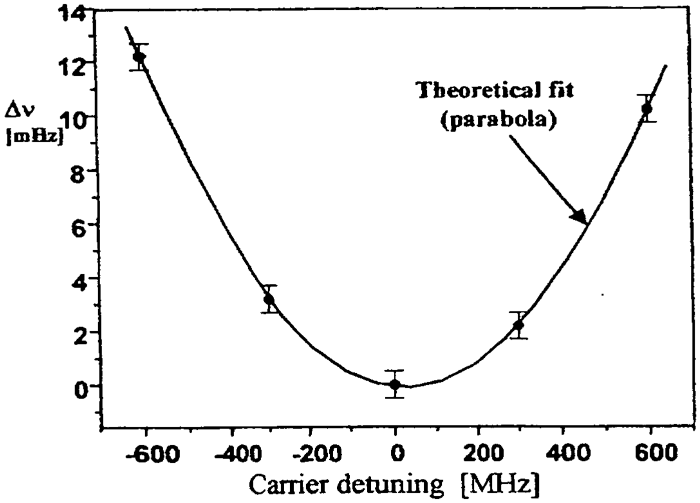

Fig. 6. Quadratic light shift as observed experimentally. The solid curve is a quadratic fit of (5) to the experimental points with $m = 0.5$.

the origin of the frequency shift axis was set in such a way as to fit (6) to the experimental data with $\Delta\nu = 0$ at $m = 2.405$.

The line width is given by the expression

$$\Delta\nu_{1/2} = (1/\pi)\left(\gamma_2 + \omega_{\mathrm{Rsb}}^2/\Gamma^*\right) \tag{8}$$

where ω_{Rsb} is the sideband Rabi frequency. The decay of the excited P state Γ^* was evaluated from the measured optical line widths to be approximately $4\times10^9 s^{-1}$. In such a case, ω_{RL} is evaluated to be of the order of $2\pi \times 1.33 \times 10^6 s^{-1}$. The calculated light shift is

$$\frac{\Delta\omega_{\mathrm{LS}}}{\omega_{\mu'\mu}} = 2.1 \times 10^{-8}\Theta(m).$$

A fit through the experimental points gives

$$\frac{\Delta\omega_{\mathrm{LS}}}{\omega_{\mu'\mu}} = 5.5 \times 10^{-8}\Theta(m)$$

with a value falling to zero for $m = 2.4$. In view of the simple model used, the agreement has to be considered satisfactory.

B. Quadratic Light Shift

The light shift was also measured for a fixed laser power and index of modulation as a function of laser frequency. The results are shown in Fig. 6. The experimental conditions were laser modulation frequency, 4.6 GHz; relaxation rate, $\gamma_2 = 330 s^{-1}$ ($T_{\mathrm{cell}} = 40°\mathrm{C}$); modulation index, $m = 0.5$, which sets $\xi = 3.6$; laser power adjusted to obtain a maser line width, 900 Hz. The origin of the maser frequency shift axis $\Delta\nu$ was set arbitrarily equal to zero for the purpose of comparison with the theoretical quadratic term of (5). This origin, however, depends on the index of modulation as demonstrated in Fig. 5.

A fit to the experimental data gives

$$\frac{\Delta\omega_{\mathrm{LS}}}{\omega_{\mu'\mu}} = 6.0 \times 10^{-15}/(\mathrm{MHz})^2.$$

Using the experimental conditions mentioned previously, the Rabi laser angular frequency ω_{RL} is evaluated to be equal to $2\pi \times 2$ MHz. Under these conditions, the light shift calculated from (5) is

$$\frac{\Delta\omega_{\mathrm{LS}}}{\omega_{\mu'\mu}} = 2.1 \times 10^{-15}/(\mathrm{MHz})^2.$$

This is a factor three smaller than the experimental value. It is possible that the simple three level model used does not represent well the experimental situation in the case of the D_2 transition; measurements could be done with the D_1 rather than the D_2 transition. In that case, the optical spectrum would be simpler and closer to the theoretical three-level model used.

IV. Conclusion

In the present paper, we have shown that the light shift in the CPT maser originates from two contributions: a power light shift independent of laser tuning and a quadratic light shift dependent on the square of the frequency tuning of the laser. These contributions can be explained qualitatively by means of a simple three-level model. The light shift originates from the combined effect of the residual carrier and the sidebands on the optical transitions that are part of the CPT Λ scheme. It was found that both contributions may be made to vanish with the proper modulation index. Finally, it was found that the data are in fairly good agreement with calculations made using a simple three-level model of the lower states of the cesium atom. To have a more detailed and precise comparison with the theory, an atomic structure closest to the three-level model is required. The D_1 transition is the natural candidate for its highest hyperfine splitting in the excited states and for the absence of the cycling transitions.

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